

Mass-Deformed Super Yang-Mills Theories from M2-Branes with Flux

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Abstract

We consider $(2+1)$ -dimensional mass-deformed SYM theories and their M-theory origin. These are obtained from MP Higgsing of ABJM theory with constant flux and fixing the mass terms via supersymmetry completion. Depending on the choice of the flux, we obtain $\mathcal{N} = 1$, $\mathcal{N} = 2$, and $\mathcal{N} = 4$ mass-deformed SYM theories. For each of these cases we solve the vacuum equation and obtain the fuzzy two ellipsoid solution for the first two cases. We also discuss the D-brane interpretation of the obtained mass-deformed SYM theories.

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1 Introduction

Proposal of the theories describing the low energy dynamics of multiple M2-branes has drastically improved our understanding of M-theory [1, 2, 3]. Subsequently, various deformations of Bagger-Lambert, Gustavsson (BLG) and Aharony-Bergman-Jafferis-Maldacena (ABJM) theories have been discussed, including maximal supersymmetry preserving mass-deformation of BLG theory [4, 5] and ABJM theory [6, 7], higher derivative corrections to BLG theory [8], addition of matter multiplets in fundamental representations in ABJM theory [9, 10], introduction of the Wess-Zumino (WZ) type couplings to the background form fields [11, 12, 13, 14, 15, 16, 17], and so on. One noteworthy aspect is the fact that the dimensional reduction of the theory of multiple M2-branes via the Mukhi-Papageorgakis (MP) Higgsing procedure [18] provides the description of low energy dynamics of multiple D2-branes, the $\mathcal{N} = 8$ super Yang-Mills (SYM) theory in $(2+1)$ -dimensions. Since the MP Higgsing of the undeformed theories gives the $(2+1)$ -dimensional SYM theory, it is intriguing to investigate which of the deformations allow the Higgsing procedure for dimensional reduction and what are their resultant theories.

In this paper we are interested in the supersymmetry preserving mass-deformations in the ABJM theory, which are generated by turning on a transverse constant four-form field strength and the dual seven-form field strength [13, 16, 14]. Though it is natural to begin with the maximal supersymmetry preserving mass-deformation, the presence of quadratic mass terms for all the transverse scalar fields makes the bosonic potential have no flat direction. Since the MP Higgsing

procedure is not applicable to this case, we shall start with the ABJM theory deformed by a WZ-type coupling to a six-form gauge field with an arbitrary constant seven-form field strength. In general this deformation breaks supersymmetry, however, the MP Higgsing procedure is applicable for this model and the results is a (2+1)-dimensional Yang-Mills matter Lagrangian involving the Myers coupling to five-form gauge field [19]. From the side of type IIA string theory, we may have some mass-deformed (2+1)-dimensional SYM theories which can be obtained through a circle compactification of one world-volume direction in the Polchinski-Strasler $\mathcal{N} = 1^*$ and $\mathcal{N} = 2^*$ theories in (3+1)-dimensions [20]. Keeping these two sets of (2+1)-dimensional Yang-Mills theories in mind, one may expect to preserve some supersymmetry for the former theory and reproduce an equivalent of the later theory. We show that preservation of supersymmetries will fix the values of the nonvanishing components of the five-form gauge field as well as the mass terms for the fermionic and the scalar fields. Depending on the choice of the flux, we obtain $\mathcal{N} = 2$ and $\mathcal{N} = 4$ mass-deformed SYM theories, which are linked with the $\mathcal{N} = 1^*$ and $\mathcal{N} = 2^*$ theories, as well as a $\mathcal{N} = 1$ theory. After fixing the form of the five-form gauge field using supersymmetry invariance in type IIA string theory, we combined this result with that of the MP Higgsing procedure to determine the form of the corresponding six-form gauge field in M-theory. The resulting six-form gauge field is different from the one which generates the maximal supersymmetry preserving mass-deformation in ABJM theory.

The remaining part of the paper is organized as follows. In section 2 we apply the MP Higgsing procedure to the original ABJM theory and verify that this results in a supersymmetry enhancement and reproduces the $\mathcal{N} = 8$ SYM in (2+1)-dimensions. In section 3 we apply the Higgsing procedure to the WZ-type coupling for a constant seven-form field strength and obtain the corresponding Myers coupling to a five form-field in type IIA string theory. In section 4 we use the Myers coupling of the type determined in section 3 and obtain the mass-deformed SYM theories preserving $\mathcal{N} = 1$, $\mathcal{N} = 2$, or $\mathcal{N} = 4$ supersymmetries. For each of these cases we will solve the vacuum equation and obtain the fuzzy two ellipsoid solution in the first two cases. We also discuss the D-brane interpretation of the obtained mass-deformed SYM theories. In section 5 we invert the results of the Higgsing procedure and identify the possible M-theory origin of the fluxes which generate supersymmetry preserving mass-deformations in type IIA string theory. Section 6 is devoted to conclusions and future research directions.

2 Higgsing of the ABJM Theory

Based on the BLG theory, Mukhi and Papageorgakis established a Higgsing procedure [18], which reduces the theory describing multiple M2-branes to theory of multiple D2-branes. An extension

to the ABJM theory with $U(N) \times U(N)$ gauge group was made in Ref. [21] and they obtained (2+1)-dimensional $\mathcal{N} = 8$ SYM theory with $U(N)$ gauge group. In their setup, the Higgsing procedure also gives a Lagrangian of a free scalar field which is decoupled from the other fields. In this section we recapitulate the calculation that the application of MP Higgsing procedure to the same ABJM theory in the setup of Ref. [16] reproduce the same SYM theory without the decoupled free scalar field.

The ABJM action [3] is given by a Chern-Simons matter theory with $\mathcal{N} = 6$ supersymmetry and $U(N) \times U(N)$ gauge symmetry,

$$S = \int d^3x (\mathcal{L}_0 + \mathcal{L}_{\text{CS}} + \mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{bos}}), \quad (2.1)$$

where

$$\mathcal{L}_0 = \text{tr} \left(-D_\mu Y_A^\dagger D^\mu Y^A + i\Psi^{\dagger A} \gamma^\mu D_\mu \Psi_A \right), \quad (2.2)$$

$$\mathcal{L}_{\text{CS}} = \frac{k}{4\pi} \epsilon^{\mu\nu\rho} \text{tr} \left(A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho - \hat{A}_\mu \partial_\nu \hat{A}_\rho - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right), \quad (2.3)$$

$$\begin{aligned} \mathcal{L}_{\text{ferm}} = & -\frac{2\pi i}{k} \text{tr} \left(Y_A^\dagger Y^A \Psi^{\dagger B} \Psi_B - Y^A Y_A^\dagger \Psi_B \Psi^{\dagger B} + 2Y^A Y_B^\dagger \Psi_A \Psi^{\dagger B} - 2Y_A^\dagger Y^B \Psi^{\dagger A} \Psi_B \right. \\ & \left. + \epsilon^{ABCD} Y_A^\dagger \Psi_B Y_C^\dagger \Psi_D - \epsilon_{ABCD} Y^A \Psi^{\dagger B} Y^C \Psi^{\dagger D} \right), \end{aligned} \quad (2.4)$$

$$\begin{aligned} \mathcal{L}_{\text{bos}} = & \frac{4\pi^2}{3k^2} \text{tr} \left(Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + 4Y_A^\dagger Y^B Y_C^\dagger Y^A Y_B^\dagger Y^C \right. \\ & \left. - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right). \end{aligned} \quad (2.5)$$

The four complex scalar fields Y^A ($A = 1, 2, 3, 4$) represent the eight directions X^I ($I = 1, \dots, 8$) transverse to the M2-branes,

$$Y^A = X^A + iX^{A+4}. \quad (2.6)$$

Let us take into account Higgsing of the bosonic sector of this Lagrangian. For completeness we briefly summarize the Higgsing procedure of the Ref. [16] including the fermionic sector. The first step is breaking the $U(N) \times U(N)$ gauge symmetry down to $U(N)$, so that the scalar fields are in the adjoint representation of the unbroken $U(N)$. As a result, the transverse scalars X^I can be split into their trace and traceless part as

$$X^I = \tilde{X}^I + i\hat{X}^I = \tilde{X}_0^I T^0 + i\hat{X}_\alpha^I T^\alpha, \quad (2.7)$$

where T^0 and T^α ($\alpha = 1, \dots, N^2 - 1$) are the generators of $U(1)$ and $SU(N)$, respectively. Then the covariant derivatives of the complex scalars become

$$D_\mu Y^A = \tilde{D}_\mu X^A + i\tilde{D}_\mu X^{A+4} + i\{A_\mu^-, X^A + iX^{A+4}\}, \quad (2.8)$$

where $\tilde{D}_\mu X = \partial_\mu X + i[A_\mu^+, X]$ and $A_\mu^\pm = \frac{1}{2}(A_\mu \pm \hat{A}_\mu)$.

The next step of the Higgsing procedure is to turn on vacuum expectation value v for the trace part of one of the complex scalar fields,

$$\begin{aligned} Y^A &= \frac{v}{2} T^0 \delta^{A4} + X^A + iX^{A+4} \\ &= \frac{v}{2} T^0 \delta^{A4} + \tilde{X}^A + i\tilde{X}^{A+4}. \end{aligned} \quad (2.9)$$

In the second line we introduced eight Hermitian scalar fields in the adjoint representation of the unbroken $U(N)$ as

$$\tilde{X}^A = \tilde{X}^A - \hat{X}^{A+4}, \quad \tilde{X}^{A+4} = \tilde{X}^{A+4} + \hat{X}^A. \quad (2.10)$$

Now we take a double scaling limit of large v and large Chern-Simons level k with finite v/k . To the leading order in $1/v$ the covariant derivatives (2.8) under a gauge choice $A_\mu^- \rightarrow A_\mu^- - \frac{1}{v} \tilde{D}_\mu (\tilde{X}^8 + \hat{X}^4)$ become

$$\begin{aligned} D_\mu Y^4 &= \tilde{D}_\mu (\tilde{X}^4 - \hat{X}^8) + iv[A_\mu^- + \frac{1}{v}(\tilde{D}_\mu (\tilde{X}^8 + \hat{X}^4))] = \tilde{D}_\mu \tilde{X}^4 + ivA_\mu^-, \\ D_\mu Y^a &= \tilde{D}_\mu [(\tilde{X}^a + i\hat{X}^a) + i(\tilde{X}^{a+4} + i\hat{X}^{a+4})] = \tilde{D}_\mu \tilde{X}^a + i\tilde{D}_\mu \tilde{X}^{a+4}, \quad a = 1, 2, 3. \end{aligned} \quad (2.11)$$

Then the gauge field A_μ^- is an auxiliary field in the resulting action and can be integrated out. This completes the Higgsing of the bosonic part of the ABJM theory, which results in the Lagrangian

$$\mathcal{L}_{\text{bos}} = \frac{1}{g^2} \text{tr} \left(-\tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i - \frac{1}{2} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} [\tilde{X}^i, \tilde{X}^j]^2 \right), \quad i, j = 1, \dots, 7, \quad (2.12)$$

where $\tilde{F}_{\mu\nu}$ is the field strength of the gauge field A_μ^+ and $g = \frac{2\pi v}{k}$ is the Yang-Mills coupling. In the last step we have rescaled the scalar fields as $\tilde{X}^i \rightarrow \frac{1}{g} \tilde{X}^i$.

In order to perform the Higgsing procedure in the fermionic sector we start by splitting the complex fermionic fields as

$$\Psi_A = \psi_A + i\psi_{A+4}. \quad (2.13)$$

Once the $U(N) \times U(N)$ gauge symmetry is broken down to $U(N)$, the fermions are also in the adjoint representation of the unbroken $U(N)$. Then we can split the trace and the traceless parts of ψ_A and ψ_{A+4} ,

$$\begin{aligned} \psi_A &= \tilde{\psi}_A + i\hat{\psi}_A = (\tilde{\psi}_A)_0 T^0 + i(\hat{\psi}_A)_\alpha T^\alpha, \\ \psi_{A+4} &= \tilde{\psi}_{A+4} + i\hat{\psi}_{A+4} = (\tilde{\psi}_{A+4})_0 T^0 + i(\hat{\psi}_{A+4})_\alpha T^\alpha. \end{aligned} \quad (2.14)$$

By introducing eight Hermitian fermionic fields in the adjoint representations,

$$\tilde{\psi}_A = \check{\psi}_A - \hat{\psi}_{A+4}, \quad \tilde{\psi}_{A+4} = \check{\psi}_{A+4} + \hat{\psi}_A, \quad (2.15)$$

we rewrite the fermions (2.13) as

$$\Psi_A = \check{\psi}_A + i\hat{\psi}_A + i(\check{\psi}_{A+4} + i\hat{\psi}_{A+4}) = \check{\psi}_A + i\tilde{\psi}_{A+4}. \quad (2.16)$$

In the double scaling limit, the covariant derivatives of the fermionic field to the leading order in $1/v$ become

$$D_\mu \Psi_A = \tilde{D}_\mu \left[\check{\psi}_A + i\hat{\psi}_A + i(\check{\psi}_{A+4} + i\hat{\psi}_{A+4}) \right] = \tilde{D}_\mu \check{\psi}_A + i\tilde{D}_\mu \tilde{\psi}_{A+4}. \quad (2.17)$$

Substituting (2.16)–(2.17) into the ABJM Lagrangian and rescaling the fermions as $\tilde{\psi}_r \rightarrow \frac{1}{g}\tilde{\psi}_r$, we obtain the fermionic kinetic term from (2.2),

$$\text{tr}(i\Psi^\dagger \gamma^\mu D_\mu \Psi_A) = \frac{1}{g^2} \text{tr}(i\tilde{\psi}_r \gamma^\mu \tilde{D}_\mu \tilde{\psi}_r), \quad r = 1, \dots, 8. \quad (2.18)$$

Similarly the Higgsing of the ABJM fermionic potential (2.4) produces the following Yukawa type coupling

$$\mathcal{L}_{\text{Yukawa}} = -\frac{1}{g^2} \text{tr}\{\Gamma_i^{rs} \tilde{\psi}_r [\tilde{X}^i, \tilde{\psi}_s]\}, \quad i = 1, \dots, 7. \quad (2.19)$$

Here Γ_i 's are turned out to be the 7-dimensional Euclidean gamma matrices satisfying

$$\{\Gamma_i, \Gamma_j\} = -2\delta_{ij}. \quad (2.20)$$

In the current notation they are given by

$$\begin{aligned} \Gamma_1 &= -(i\sigma_2 \otimes \Delta_2 + \sigma_1 \otimes \Delta_3), & \Gamma_2 &= -(i\sigma_2 \otimes \Delta_4 + \sigma_1 \otimes \Delta_5), \\ \Gamma_3 &= -(i\sigma_2 \otimes \Delta_6 + \sigma_1 \otimes \Delta_7), & \Gamma_4 &= i\sigma_2 \otimes \Delta_1, & \Gamma_5 &= -(\mathbb{I} \otimes \Delta_8 - \sigma_2 \otimes \Delta_3), \\ \Gamma_6 &= -(\mathbb{I} \otimes \Delta_9 - \sigma_2 \otimes \Delta_5), & \Gamma_7 &= -(\mathbb{I} \otimes \Delta_{10} - \sigma_2 \otimes \Delta_7), \end{aligned} \quad (2.21)$$

where $\sigma_{1,2}$ are the first and the second Pauli matrices, while Δ_k are 4×4 matrices,

$$\begin{aligned} \Delta_1^{pq} &= \delta^{pq} - 2\delta^{p4}\delta^{q4}, & \Delta_2^{pq} &= \delta^{p4}\delta^{q1} + \delta^{p1}\delta^{q4}, & \Delta_3^{pq} &= \delta^{p2}\delta^{q3} - \delta^{p3}\delta^{q2}, \\ \Delta_4^{pq} &= \delta^{p4}\delta^{q2} + \delta^{p2}\delta^{q4}, & \Delta_5^{pq} &= \delta^{p3}\delta^{q1} - \delta^{p1}\delta^{q3}, & \Delta_6^{pq} &= \delta^{p4}\delta^{q3} + \delta^{p3}\delta^{q4}, \\ \Delta_7^{pq} &= \delta^{p1}\delta^{q2} - \delta^{p2}\delta^{q1}, & \Delta_8^{pq} &= \delta^{p4}\delta^{q1} - \delta^{p1}\delta^{q4}, & \Delta_9^{pq} &= \delta^{p4}\delta^{q2} - \delta^{p2}\delta^{q4}, \\ \Delta_{10}^{pq} &= \delta^{p4}\delta^{q3} - \delta^{p3}\delta^{q4}. \end{aligned} \quad (2.22)$$

Collecting the results in (2.12), (2.18), and (2.19), we obtain the Lagrangian of $(2+1)$ -dimensional $\mathcal{N} = 8$ SYM with $U(N)$ gauge symmetry,

$$\mathcal{L}_{\text{SYM}}^{\mathcal{N}=8} = \frac{1}{g^2} \text{tr} \left(-\frac{1}{2} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \tilde{D}_\mu \tilde{X}^i \tilde{D}^\mu \tilde{X}^i + i \tilde{\psi}_r \gamma^\mu \tilde{D}_\mu \tilde{\psi}_r + \frac{1}{2} [\tilde{X}^i, \tilde{X}^j]^2 - \Gamma_i^{rs} \tilde{\psi}_r [\tilde{X}^i, \tilde{\psi}_s] \right). \quad (2.23)$$

The supersymmetry variations of the gauge and the matter fields are given by

$$\begin{aligned} \delta_\epsilon A_\mu^+ &= i \tilde{\epsilon}^r \gamma_\mu \tilde{\psi}_r, \\ \delta_\epsilon \tilde{X}^i &= i \Gamma_i^{rs} \tilde{\epsilon}_r \tilde{\psi}_s, \\ \delta_\epsilon \tilde{\psi}_r &= i \tilde{F}_{\mu\nu} \sigma^{\mu\nu} \tilde{\epsilon}_r + \Gamma_i^{rs} \gamma^\mu \tilde{\epsilon}_s D_\mu \tilde{X}^i - \Gamma_{ij}^{rs} \tilde{\epsilon}_s [\tilde{X}^i, \tilde{X}^j], \end{aligned} \quad (2.24)$$

where $\tilde{\epsilon}_r = \tilde{\epsilon}^r$ and

$$\sigma^{\mu\nu} = -\frac{i}{4} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \quad \Gamma_{ij} = \frac{i}{4} (\Gamma_i \Gamma_j - \Gamma_j \Gamma_i). \quad (2.25)$$

Six of these eight supersymmetries are inherited from the six supersymmetries of the ABJM theory. This can be easily identified by setting $\tilde{\epsilon}_4 = \tilde{\epsilon}_8 = 0$ in (2.24). In addition to these six supersymmetries, additional two supersymmetries arise as a consequence of the breaking of the gauge symmetry in the MP Higgsing procedure which moves the M2-branes away from the orbifold singularity.

3 Higgsing of WZ-type Coupling with Constant Flux

For the ABJM theory, there is mass deformation which preserves $\mathcal{N} = 6$ supersymmetry [6, 7]. The origin of this deformation is a WZ-type coupling to constant transverse four-form field strength which is dual to constant seven-form field strength. This coupling with a particular choice of the constant field strength can be identified with the quartic self interaction term of transverse scalars [16], while the quadratic mass term is understood as a result of the backreaction of the flux on the geometry [14]. In this section we will consider more generic constant transverse four-form and the dual seven-form field strengths and reduce the corresponding WZ-type coupling to type IIA string theory via the Higgsing procedure.

First let us consider the WZ-type coupling for the three-form field C_3 . In the presence of a constant transverse four-form field strength F_4 the components of the corresponding C_3 are independent of the worldvolume coordinates and are at most linear in the transverse coordinates. More precisely, $C_{\mu\nu\rho}$, $C_{\mu\nu A}$, $C_{\mu AB}$, and $C_{\mu A\bar{B}}$ are all constants while C_{ABC} and $C_{AB\bar{C}}$ are linear in the transverse coordinates.¹ We set the constant components to zero by gauge freedom of the

¹We employ the same index notation as in Ref. [16].

three-form gauge field ($\delta C_3 = d\Lambda_2$). Then the gauge invariant WZ-type coupling for this particular choice of the three-form gauge field can be read from the equation (2.3) of Ref. [16],

$$S_C^{(3)} = \lambda \int d^3x \frac{1}{3!} \epsilon^{\mu\nu\rho} \text{tr} \left[C_{A\bar{B}C} D_\mu Y^A D_\rho Y_B^\dagger D_\nu Y^C + (\text{c.c.}) \right], \quad (3.26)$$

where $\lambda = 2\pi l_P^{3/2}$ with Planck length l_P .

The dual seven-form field strength F_7 is given by

$$F_7 = *F_4 + \frac{1}{2} C_3 \wedge F_4. \quad (3.27)$$

This implies, in the presence of the constant transverse F_4 the components of F_7 with all indices in the transverse directions are linear in the scalar fields while the remaining components are constants. Keeping this in mind, the gauge invariant WZ-type coupling for the corresponding six-form gauge field C_6 can be read from the equation (2.8) of Ref. [16],

$$\begin{aligned} S_C^{(6)} = -\frac{\pi}{k\lambda} \int d^3x \frac{1}{3!} \epsilon^{\mu\nu\rho} \{ \text{Tr} \} & \left[C_{\mu\nu\rho A\bar{B}\bar{C}} \beta_C^{AB} + 3\lambda^2 (C_{\mu ABC\bar{D}\bar{E}} D_\nu Y^A D_\rho Y_D^\dagger \beta_E^{BC} \right. \\ & + C_{\mu A\bar{B}\bar{C}\bar{D}\bar{E}} D_\nu Y_C^\dagger D_\rho Y_D^\dagger \beta_E^{AB}) + \lambda^3 (C_{ABCD\bar{E}\bar{F}} D_\mu Y^A D_\nu Y^B D_\rho Y_E^\dagger \beta_F^{CD} \\ & \left. + C_{AB\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y^A D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta_F^{BC} + C_{A\bar{B}\bar{C}\bar{D}\bar{E}\bar{F}} D_\mu Y_C^\dagger D_\nu Y_D^\dagger D_\rho Y_E^\dagger \beta_F^{AB}) + (\text{c.c.}) \right], \end{aligned} \quad (3.28)$$

where $\beta_C^{AB} \equiv \frac{1}{2}(Y^A Y_C^\dagger Y^B - Y^B Y_C^\dagger Y^A)$ and we have set constant components of C_6 to zero using gauge freedom.

Now we apply the MP Higgsing to the WZ-type couplings (3.26) and (3.28). The three-form coupling in (3.26) and all the six-form couplings in (3.28) except for the first term produce higher order in α' after the Higgsing. Since we are interested in the terms which are in the lowest order in α' , we will neglect those higher terms and take into account only the following WZ-type coupling

$$S_C^{(6)} = -\frac{\pi}{\lambda k} \int d^3x \frac{1}{3!} \epsilon^{\mu\nu\rho} \text{tr} [C_{\mu\nu\rho A\bar{B}\bar{C}} \beta_C^{AB} + C_{\mu\nu\rho A\bar{B}\bar{C}}^\dagger (\beta_C^{AB})^\dagger]. \quad (3.29)$$

The six-form gauge fields which are linear in the transverse scalars are given by

$$C_{\mu\nu\rho A\bar{B}\bar{C}} = -2\lambda \epsilon_{\mu\nu\rho} T_{A\bar{B}\bar{C}\bar{D}} Y_D^\dagger, \quad C_{\mu\nu\rho A\bar{B}\bar{C}}^\dagger = -2\lambda \epsilon_{\mu\nu\rho} T_{A\bar{B}\bar{C}\bar{D}}^* Y^D = -2\lambda \epsilon_{\mu\nu\rho} T_{CD\bar{A}\bar{B}} Y^D, \quad (3.30)$$

where the complex-valued parameters $T_{A\bar{B}\bar{C}\bar{D}}$ are antisymmetric in the first two indices as well as the last two indices.

The Higgsing procedure for the WZ-type coupling was established in more general setting in Ref. [16]. Along the same line the Higgsing of the (3.29) results in the following Myers coupling,

$$\tilde{S}_C^{(5)} = -\frac{i\pi v}{\lambda k} \int d^3x \frac{1}{3!} \epsilon^{\mu\nu\rho} \text{tr} (\tilde{C}_{\mu\nu\rho ij} [\tilde{X}^i, \tilde{X}^j]), \quad i, j = 1, \dots, 7. \quad (3.31)$$

Here \tilde{X}^i 's are defined in (2.10) and the R-R five-form fields $\tilde{C}_{\mu\nu\rho ij}$ are identified as

$$\begin{aligned}\tilde{C}_{\mu\nu\rho ab} &= -\frac{i}{4}(C_{\mu\nu\rho a4\bar{b}} - C_{\mu\nu\rho a4\bar{b}}^\dagger - C_{\mu\nu\rho b4\bar{a}} + C_{\mu\nu\rho b4\bar{a}}^\dagger + C_{\mu\nu\rho ab\bar{4}} - C_{\mu\nu\rho ab\bar{4}}^\dagger), \\ \tilde{C}_{\mu\nu\rho a4} &= -\frac{i}{2}(C_{\mu\nu\rho a4\bar{4}} - C_{\mu\nu\rho a4\bar{4}}^\dagger), \quad \tilde{C}_{\mu\nu\rho 4a+4} = -\frac{1}{2}(C_{\mu\nu\rho a4\bar{4}} + C_{\mu\nu\rho a4\bar{4}}^\dagger), \\ \tilde{C}_{\mu\nu\rho ab+4} &= -\frac{1}{4}(C_{\mu\nu\rho a4\bar{b}} + C_{\mu\nu\rho a4\bar{b}}^\dagger + C_{\mu\nu\rho b4\bar{a}} + C_{\mu\nu\rho b4\bar{a}}^\dagger - C_{\mu\nu\rho ab\bar{4}} - C_{\mu\nu\rho ab\bar{4}}^\dagger), \\ \tilde{C}_{\mu\nu\rho a+4b+4} &= -\frac{i}{4}(C_{\mu\nu\rho a4\bar{b}} - C_{\mu\nu\rho a4\bar{b}}^\dagger - C_{\mu\nu\rho b4\bar{a}} + C_{\mu\nu\rho b4\bar{a}}^\dagger - C_{\mu\nu\rho ab\bar{4}} + C_{\mu\nu\rho ab\bar{4}}^\dagger),\end{aligned}\tag{3.32}$$

where $a, b = 1, 2, 3$. For the case of the linear six-form gauge field in (3.30), the corresponding R-R five-form gauge field is

$$\tilde{C}_{\mu\nu\rho ij} = -2\lambda\epsilon_{\mu\nu\rho}\tilde{T}_{ijk}\tilde{X}^k,\tag{3.33}$$

where \tilde{T}_{ijk} are antisymmetric real-valued parameters. Using (3.30), (3.32), and (3.33) in the action(3.31) and rescaling $\tilde{X}^i \rightarrow \frac{1}{g}\tilde{X}^i$, we obtain Myers coupling for a constant R-R five-form field in type IIA string theory [19],

$$\tilde{S}_C^{(5)} = \frac{i}{g^2} \int d^3x \operatorname{tr}(\tilde{T}_{ijk}\tilde{X}^i[\tilde{X}^j, \tilde{X}^k])\tag{3.34}$$

with

$$\begin{aligned}\tilde{T}_{ab4} &= \tilde{T}_{4a+4b+4} = -\frac{i}{2}(T_{a4\bar{b}\bar{4}} - T_{b4\bar{a}\bar{4}}), \quad \tilde{T}_{a4b+4} = \frac{1}{2}(T_{a4\bar{b}\bar{4}} + T_{b4\bar{a}\bar{4}}), \\ \tilde{T}_{abc} &= -\frac{i}{4}(T_{a4\bar{b}\bar{c}} + T_{c4\bar{a}\bar{b}} + T_{b4\bar{c}\bar{a}} - T_{bc\bar{a}\bar{4}} - T_{ab\bar{c}\bar{4}} - T_{cab\bar{4}}), \\ \tilde{T}_{abc+4} &= -\frac{1}{4}(T_{a4\bar{b}\bar{c}} - T_{c4\bar{a}\bar{b}} + T_{b4\bar{c}\bar{a}} + T_{bc\bar{a}\bar{4}} - T_{ab\bar{c}\bar{4}} + T_{cab\bar{4}}), \\ \tilde{T}_{ab+4c+4} &= \frac{i}{4}(T_{a4\bar{b}\bar{c}} - T_{c4\bar{a}\bar{b}} - T_{b4\bar{c}\bar{a}} - T_{bc\bar{a}\bar{4}} + T_{ab\bar{c}\bar{4}} + T_{cab\bar{4}}), \\ \tilde{T}_{a+4b+4c+4} &= -\frac{1}{4}(T_{a4\bar{b}\bar{c}} + T_{c4\bar{a}\bar{b}} + T_{b4\bar{c}\bar{a}} + T_{bc\bar{a}\bar{4}} + T_{ab\bar{c}\bar{4}} + T_{cab\bar{4}}).\end{aligned}\tag{3.35}$$

In section 4, we will consider the $\mathcal{N} = 8$ SYM theory discussed in section 2 and deform it by the Myers coupling (3.34). In general such deformation breaks the supersymmetry. However, if one also include an appropriate quadratic mass term and turn only some particular nonvanishing components of \tilde{T}_{ijk} , it is possible to preserve some supersymmetries.

4 Mass-deformations of (2+1)-dimensional SYM

In section 2 we have seen that the Higgsing of the undeformed ABJM theory led to the $\mathcal{N} = 8$ SYM theory in (2+1)-dimensions, with $U(N)$ gauge symmetry, however, the maximal supersymmetric

ABJM theory with the quadratic mass term does not have any flat direction and as a result the MP Higgsing procedure cannot be applied to this case. In $(3+1)$ -dimensions some mass-deformed SYM theories have already been constructed [20], where the origin of deformations in $\mathcal{N} = 1^*$ and $\mathcal{N} = 2^*$ SYM theories were interpreted as the Myers couplings of D3-branes with constant background flux in type IIB string theory. We naturally expect that dimensional reduction of these theories results in $(2+1)$ -dimensional mass-deformed SYM theories with certain amount of supersymmetry.

In the framework of AdS/CFT correspondence, $(2+1)$ -dimensional mass-deformed SYM theories have been studied in Refs. [22, 23, 24, 25]. Along the same line with the $(3+1)$ -dimensional SYM theories [20], mass term for the fermionic fields was turned on in the $(2+1)$ -dimensional field theory side and the corresponding background flux in type IIA supergravity was found. In the presence of the background flux, the D2-branes are polarized into NS5-branes when all the fermions are massive or polarized into D4-branes when one of the fermions is left massless. Based on these brane interpretations, the M-theory origin of the mass-deformed SYM theories was also discussed. The story and its M-theory interpretation are not complete because the Lagrangian with supersymmetry transformation rules was not explicitly written and because the Lagrangian formulation of multiple M2-branes was not known at the time. We will fill the gaps of the scenario in this and the subsequent sections. Guided by supersymmetry invariance, we obtain mass-deformations of $(2+1)$ -dimensional SYM theories with $\mathcal{N} = 1$, $\mathcal{N} = 2$, and $\mathcal{N} = 4$ supersymmetries. The number of supersymmetries depends on the choice of the constant background flux configuration generating the deformations in type IIA string theory.

We start by considering a configuration of multiple D2-branes on a background of constant transverse four-form field strength F_{ijkl} in the absence of NS-NS two-form field. Then the dual six-form field strength is given by

$$\frac{1}{6!}F_{\mu\nu\rho ijk} = \frac{1}{4!}\epsilon_{\mu\nu\rho ijk}{}^{i'j'k'l'}F_{i'j'k'l'}, \quad (4.36)$$

and in the symmetric gauge

$$\tilde{C}_{\mu\nu\rho ij} = \tilde{\lambda}F_{\mu\nu\rho ijk}\tilde{X}^k = \tilde{\lambda}\epsilon_{\mu\nu\rho}\tilde{T}_{ijk}\tilde{X}^k, \quad (4.37)$$

where $\tilde{\lambda} = 2\pi l_s^2$. The Myers coupling for this five-form gauge field (4.37) gives cubic self-interaction terms between the transverse scalar fields

$$\mathcal{L}_{XXX} = \frac{i}{g^2}\text{tr}(\tilde{T}_{ijk}\tilde{X}^i[\tilde{X}^j, \tilde{X}^k]), \quad (4.38)$$

while the Myers coupling for the three-form gauge field is either a constant term or higher α' corrections, which we do not take into account in the low energy effective theory of our interest.

By calculating the backreaction of the constant four-form field strengths on the geometry, one may obtain the following quadratic term

$$\mathcal{L}_{XX} = -\frac{1}{g^2} M_{ij} \text{tr}(\tilde{X}^i \tilde{X}^j), \quad (4.39)$$

where the specific form of the mass matrix M_{ij} is controlled by the supersymmetry of the mass-deformed theory.

4.1 $\mathcal{N} = 1$

In this subsection we will determine a choice of the constant six-form field strength which breaks all the supersymmetries of the $\mathcal{N} = 8$ undeformed SYM but preserves $\mathcal{N} = 1$. In this case we can set all the supersymmetry parameters in (2.24) to zero except for one parameter. For instance we choose nonvanishing $\tilde{\epsilon}_1$ and then set $\tilde{\epsilon}_r = \epsilon \delta_{1r}$. Naturally we group the fermionic fields as

$$\tilde{\lambda} \equiv \tilde{\psi}_1, \quad \text{and} \quad \tilde{\xi}_p \equiv \tilde{\psi}_p, \quad \text{for } p = 2, \dots, 8. \quad (4.40)$$

Accordingly, the supersymmetry transformations (2.24) of the gauge and the matter fields are rewritten² as

$$\begin{aligned} \delta A_\mu^+ &= i\epsilon \gamma_\mu \tilde{\lambda}, & \delta \tilde{X}^i &= i\Gamma_i^{1p} \epsilon \tilde{\xi}_p, \\ \delta \tilde{\lambda} &= i\tilde{F}_{\mu\nu} \sigma^{\mu\nu} \epsilon, & \delta \tilde{\xi}_p &= \Gamma_i^{p1} \gamma^\mu \epsilon D_\mu \tilde{X}^i - \Gamma_{ij}^{p1} \epsilon [\tilde{X}^i, \tilde{X}^j]. \end{aligned} \quad (4.41)$$

The undeformed SYM action (2.23) is manifestly invariant under such reduced $\mathcal{N} = 1$ supersymmetry.

In order to render the supersymmetry invariance of the deformed theory, we introduce the following additional supersymmetry transformation:

$$\delta' A_\mu^+ = 0, \quad \delta' \tilde{X}^i = 0, \quad \delta' \tilde{\lambda}_a = 0, \quad \delta' \tilde{\xi}_p = \mu_{pq} \Gamma_i^{q1} \epsilon \tilde{X}^i, \quad (4.42)$$

where $\mu_{pq} = \mu_p \delta_{pq}$. In this setup the μ_p 's are mass parameters for the seven massive fermionic fields $\tilde{\xi}_p$, which can have different values. When the fermionic mass term is chosen as

$$\mathcal{L}_{\text{ferm}}^\mu = -\frac{i}{g^2} \mu_{pq} \text{tr}(\tilde{\xi}_p \tilde{\xi}_q), \quad (4.43)$$

the total Lagrangian of our consideration

$$\mathcal{L}^{\mathcal{N}=1} = \mathcal{L}_{\text{SYM}}^{\mathcal{N}=8} + \mathcal{L}_{XX} + \mathcal{L}_{XXX} + \mathcal{L}_{\text{ferm}}^\mu, \quad (4.44)$$

²In this section the indices p, q, \dots run over the range $2, \dots, 8$.

becomes invariant under the total supersymmetry transformations in (4.41) and (4.42). This is possible if we choose bosonic mass matrix M_{ij} and the antisymmetric tensor \tilde{T}_{ijk} as

$$\begin{aligned} M_{ij} &= \text{diag}(\mu_8^2, \mu_7^2, \mu_6^2, \mu_5^2, \mu_4^2, \mu_3^2, \mu_2^2), \\ \tilde{T}_{ijk}(\Gamma_k)^{1p} &= \frac{1}{3}\mu_{qq'}(\Gamma_i^{1q}\Gamma_j^{q'p} - \Gamma_j^{1q}\Gamma_i^{q'p}) - \frac{2i}{3}\mu_{pq}\Gamma_{ij}^{1q}. \end{aligned} \quad (4.45)$$

From the second line of (4.45) we determine the nonvanishing components of \tilde{T}_{ijk} :

$$\begin{aligned} \tilde{T}_{145} &= -\frac{1}{3}(\mu_4 + \mu_5 + \mu_8), & \tilde{T}_{246} &= \frac{1}{3}(\mu_3 + \mu_5 + \mu_7), & \tilde{T}_{347} &= \frac{1}{3}(\mu_2 + \mu_5 + \mu_6), \\ \tilde{T}_{127} &= -\frac{1}{3}(\mu_2 + \mu_7 + \mu_8), & \tilde{T}_{136} &= \frac{1}{3}(\mu_3 + \mu_6 + \mu_8), & \tilde{T}_{235} &= \frac{1}{3}(\mu_4 + \mu_6 + \mu_7), \\ \tilde{T}_{567} &= -\frac{1}{3}(\mu_2 + \mu_3 + \mu_4). \end{aligned} \quad (4.46)$$

The $\mathcal{N} = 1$ mass-deformed SYM theory constructed here contains one massless vector multiplet $(A_\mu^+, \tilde{\lambda})$ and seven massive matter multiplets $(\tilde{X}^i, \tilde{\xi}_p)$. As mentioned previously the massive multiplets are allowed to have different masses.

Fuzzy two ellipsoid solution: From the equations (2.23), (4.38), and (4.39) we read the scalar potential of the $\mathcal{N} = 1$ mass-deformed SYM theory (4.44):

$$V(\tilde{X}) = -\frac{1}{g^2} \text{tr} \left(\frac{1}{2} [\tilde{X}^i, \tilde{X}^j]^2 + i\tilde{T}_{ijk}\tilde{X}^i[\tilde{X}^j, \tilde{X}^k] - M_{ij}\tilde{X}^i\tilde{X}^j \right), \quad (4.47)$$

where the mass matrix M_{ij} and the antisymmetric tensor \tilde{T}_{ijk} are given in (4.45)–(4.46). The classical supersymmetric vacuum equation satisfying $V(\tilde{X}_0) = \frac{\partial V(\tilde{X}_0)}{\partial \tilde{X}} = 0$ can be obtained from (4.41)–(4.42),

$$\Gamma_{ij}^{p1}[\tilde{X}^i, \tilde{X}^j] - \mu_{pq}\Gamma_i^{q1}\tilde{X}^i = 0 \quad \text{for } p = 2, 3, \dots, 8. \quad (4.48)$$

The same equation can also be derived by component field expansion of F -term equation in the $\mathcal{N} = 1$ superfield formulation as well. A nontrivial solution to this vacuum equation is the configuration of fuzzy two ellipsoid, which corresponds to D2-branes polarized into D4-branes [22, 23, 24, 25]. The fuzzy two ellipsoid can be embedded in any three-dimensional transverse space with nonvanishing flux given by (4.46) and some constraints on the masses. Without loss of generality, here we choose the directions $(1, 4, 5)$ as the embedding space of the fuzzy two ellipsoid. The ansatz is

$$\tilde{X}_0^i = \begin{cases} \alpha_i T^i & \text{for } i = 1, 4, 5, \quad (\text{no summation over } i) \\ 0 & \text{otherwise} \end{cases} \quad (4.49)$$

where T^i 's are the generators of the N -dimensional irreducible representation of $SU(2)$ and α_i are real constants of mass-dimension one. Then the \tilde{X}_0^i 's satisfy the noncommutative algebra

$$[\tilde{X}_0^i, \tilde{X}_0^j] = i\alpha_i\alpha_j\epsilon_{ijk}T^k, \quad (\text{no summation over } i, j) \quad (4.50)$$

which defines the fuzzy two ellipsoid. Inserting (4.50) into the vacuum equation (4.48), we obtain

$$\alpha_1\alpha_4 - \alpha_5\mu_4 = 0, \quad \alpha_1\alpha_5 - \alpha_4\mu_5 = 0, \quad \alpha_4\alpha_5 - \alpha_1\mu_8 = 0. \quad (4.51)$$

Assuming μ_4, μ_5, μ_8 are either all positive or all negative, we have the following set of solutions for (4.51)

$$\begin{aligned} & \alpha_1 = -\sqrt{\mu_4\mu_5}, \quad \alpha_4 = \sqrt{\mu_4\mu_8}, \quad \alpha_5 = -\sqrt{\mu_5\mu_8}, \\ \text{or } & \alpha_1 = -\sqrt{\mu_4\mu_5}, \quad \alpha_4 = -\sqrt{\mu_4\mu_8}, \quad \alpha_5 = \sqrt{\mu_5\mu_8}, \\ \text{or } & \alpha_1 = \sqrt{\mu_4\mu_5}, \quad \alpha_4 = -\sqrt{\mu_4\mu_8}, \quad \alpha_5 = -\sqrt{\mu_5\mu_8}, \\ \text{or } & \alpha_1 = \sqrt{\mu_4\mu_5}, \quad \alpha_4 = \sqrt{\mu_4\mu_8}, \quad \alpha_5 = \sqrt{\mu_5\mu_8}. \end{aligned} \quad (4.52)$$

The scale of the noncommutative space is characterized by the size of the semi-principal axes of the fuzzy two ellipsoid:

$$R_i = 2\pi\alpha' \sqrt{\frac{3}{N}\text{tr}[(\tilde{X}_0^i)^2]} = \pi\alpha' N\alpha_i \sqrt{1 - \frac{1}{N^2}}, \quad (4.53)$$

where we have used $\text{tr}(\tilde{X}_0^i)^2 = \frac{1}{12}\alpha_i^2 N(N^2 - 1)$. In the large N limit, (4.53) is understood as the size of the shell D4-brane which emerges as a result of polarization of D2-branes [23]. It is also important to note that this solution reduces to the fuzzy two sphere solution if we started with $\alpha_1 = \alpha_4 = \alpha_5$.

4.2 $\mathcal{N} = 2$

To find the $\mathcal{N} = 2$ mass-deformed SYM theory we follow a similar procedure as in the previous subsection. To that end we start by assuming that two of the supersymmetric parameters in (2.24) are nonvanishing, for instance, $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$. Then we can write the supersymmetry parameters as $\tilde{\epsilon}_r = \epsilon_1\delta_{1r} + \epsilon_2\delta_{2r}$ and group the fermionic fields as

$$\tilde{\lambda}_a \equiv \tilde{\psi}_a, \quad \text{for } a = 1, 2 \quad \text{and} \quad \tilde{\xi}_p \equiv \tilde{\psi}_p, \quad \text{for } p = 3, 4, \dots, 8. \quad (4.54)$$

Using the representation of seven-dimensional gamma matrices in (2.21), we can write the

supersymmetry transformations of the gauge and the matter fields as³

$$\begin{aligned}\delta A_\mu^+ &= i\epsilon_a \gamma_\mu \tilde{\lambda}_a, & \delta \tilde{X}^7 &= i\Gamma_7^{ab} \epsilon_a \tilde{\lambda}_b, & \delta \tilde{X}^m &= i\Gamma_m^{ap} \epsilon_a \tilde{\xi}_p, \\ \delta \tilde{\lambda}_a &= i\tilde{F}_{\mu\nu} \sigma^{\mu\nu} \epsilon_a + \Gamma_7^{ab} \gamma^\mu \epsilon_b D_\mu \tilde{X}^7 - \Gamma_{ij}^{ab} \epsilon_b [\tilde{X}^i, \tilde{X}^j], \\ \delta \tilde{\xi}_p &= \Gamma_m^{pa} \gamma^\mu \epsilon_a D_\mu \tilde{X}^m - \Gamma_{ij}^{pa} \epsilon_a [\tilde{X}^i, \tilde{X}^j].\end{aligned}\tag{4.55}$$

Note that the undeformed SYM action (2.23) is invariant under the reduced supersymmetry (4.55).

For the mass-deformed theory, along the same line with the $\mathcal{N} = 1$ SYM theory, we introduce the additional supersymmetry transformation:

$$\delta' A_\mu^+ = 0, \quad \delta' \tilde{X}^i = 0, \quad \delta' \tilde{\lambda}_a = 0, \quad \delta' \tilde{\xi}_p = \mu_{pq} \Gamma_m^{qa} \epsilon_a \tilde{X}^m,\tag{4.56}$$

and a fermionic mass term

$$\mathcal{L}_{\text{ferm}}^\mu = -\frac{i}{g^2} \mu_{pq} \text{tr}(\tilde{\xi}_p \tilde{\xi}_q).\tag{4.57}$$

As in the case of $\mathcal{N} = 1$, the mass matrix μ_{pq} is diagonal, $\mu_{pq} = \mu_p \delta_{pq}$ and its elements μ_p 's are the mass parameters for the six massive fermionic fields, which are not allowed to be all different.

From the invariance of the mass-deformed theory under (4.55)–(4.56), the mass matrix M_{mn} of six massive bosonic fields is determined as

$$M_{mn} = \text{diag}(\mu_8^2, \mu_7^2, \mu_6^2, \mu_5^2, \mu_4^2, \mu_3^2)\tag{4.58}$$

with constraints

$$\mu_4^2 = \mu_3^2, \quad \mu_6^2 = \mu_5^2, \quad \mu_8^2 = \mu_7^2.\tag{4.59}$$

The supersymmetry invariance also fixes the tensor \tilde{T}_{ijk} as

$$\begin{aligned}\tilde{T}_{ijm} \Gamma_m^{ap} &= \frac{1}{3} \mu_{qq'} (\Gamma_i^{aq} \Gamma_j^{q'p} - \Gamma_j^{aq} \Gamma_i^{q'p}) - \frac{2i}{3} \mu_{pq} \Gamma_{ij}^{aq}, \\ \tilde{T}_{ij7} \Gamma_7^{ab} &= \frac{1}{3} \mu_{pq} (\Gamma_i^{ap} \Gamma_j^{qb} - \Gamma_j^{ap} \Gamma_i^{qb}).\end{aligned}\tag{4.60}$$

From these equations we find the following nonvanishing components of the tensor \tilde{T}_{ijk} ,

$$\begin{aligned}\tilde{T}_{145} &= \frac{1}{3}(\mu_3 + \mu_6 + \mu_7), & \tilde{T}_{246} &= \frac{1}{3}(\mu_3 + \mu_5 + \mu_7), & \tilde{T}_{347} &= \frac{1}{3}(\mu_5 + \mu_6), \\ \tilde{T}_{127} &= -\frac{1}{3}(\mu_7 + \mu_8), & \tilde{T}_{136} &= -\frac{1}{3}(\mu_4 + \mu_5 + \mu_7), & \tilde{T}_{235} &= -\frac{1}{3}(\mu_3 + \mu_5 + \mu_8), \\ \tilde{T}_{567} &= -\frac{1}{3}(\mu_3 + \mu_4),\end{aligned}\tag{4.61}$$

³In this subsection we employ the following indices. The bosonic field indices m, n, \dots are used when the $i = 7$ index is excluded, the fermionic field indices a, b, \dots represent 1 or 2, the remaining fermionic field indices p, q, \dots run over $3, \dots, 8$.

where the μ_p should satisfy

$$\mu_3 + \mu_4 + \mu_5 + \mu_6 + \mu_7 + \mu_8 = 0. \quad (4.62)$$

Here we have six massive fermionic fields $\tilde{\xi}_p$ and the same number of massive bosonic fields \tilde{X}^m , which will form three massive chiral multiplets of the $\mathcal{N} = 2$ supersymmetry. In addition we have a pair of massless fermionic fields $\tilde{\lambda}_a$ and a single massless scalar \tilde{X}^7 , which together with the gauge boson A_μ^+ , form a massless vector multiplet.

Fuzzy two ellipsoid solution: Along the same line as $\mathcal{N} = 1$ theory, the classical supersymmetric vacuum equations are read from (4.55)–(4.56),

$$\Gamma_{ij}^{ab}[\tilde{X}^i, \tilde{X}^j] = 0, \quad \Gamma_{ij}^{pa}[\tilde{X}^i, \tilde{X}^j] - \mu_{pq}\Gamma_i^{qa}\tilde{X}^i = 0, \quad \text{for } a, b = 1, 2, \text{ and } p = 3, 4, \dots, 8. \quad (4.63)$$

The first equation is the component field expansion of D -term equation, while the second is the expansion of the F -term equation in $\mathcal{N} = 2$ superfield formulation. The fuzzy two ellipsoid solution to (4.63) is also obtained by using the ansatz (4.49). The D -term equation is trivially satisfied, while the F -term equation leads to

$$\begin{aligned} \alpha_1\alpha_4 + \alpha_5\mu_3 &= 0, & \alpha_1\alpha_4 - \alpha_5\mu_4 &= 0, & \alpha_1\alpha_5 - \alpha_4\mu_5 &= 0, \\ \alpha_1\alpha_5 + \alpha_4\mu_6 &= 0, & \alpha_4\alpha_5 + \alpha_1\mu_7 &= 0, & \alpha_4\alpha_5 - \alpha_1\mu_8 &= 0. \end{aligned} \quad (4.64)$$

In order to have nontrivial solution for α_i 's, we should set $\mu_4 = -\mu_3$, $\mu_6 = -\mu_5$, $\mu_8 = -\mu_7$. Then, assuming $\mu_3\mu_5 < 0$, $\mu_3\mu_7 > 0$, and $\mu_5\mu_7 < 0$, we obtain the following set of solutions

$$\begin{aligned} \alpha_1 &= -i\sqrt{\mu_3\mu_5}, \quad \alpha_4 = -\sqrt{\mu_3\mu_7}, \quad \alpha_5 = -i\sqrt{\mu_5\mu_7}, \\ \text{or } \alpha_1 &= -i\sqrt{\mu_3\mu_5}, \quad \alpha_4 = \sqrt{\mu_3\mu_7}, \quad \alpha_5 = i\sqrt{\mu_5\mu_7}, \\ \text{or } \alpha_1 &= i\sqrt{\mu_3\mu_5}, \quad \alpha_4 = \sqrt{\mu_3\mu_7}, \quad \alpha_5 = -i\sqrt{\mu_5\mu_7}, \\ \text{or } \alpha_1 &= i\sqrt{\mu_3\mu_5}, \quad \alpha_4 = -\sqrt{\mu_3\mu_7}, \quad \alpha_5 = -i\sqrt{\mu_5\mu_7}. \end{aligned} \quad (4.65)$$

The size of the fuzzy two ellipsoid and the D-brane interpretation of these solutions are the same as those of the $\mathcal{N} = 1$ case in (4.53).

4.3 $\mathcal{N} = 4$

Finally, we will find the choice of the constant flux which preserves $\mathcal{N} = 4$ supersymmetry. The detail is as in the pervious subsection. The supersymmetry transformations of the gauge and the

matter fields are⁴

$$\begin{aligned}
\delta A_\mu^+ &= i\epsilon_a \gamma_\mu \tilde{\lambda}_a, & \delta \tilde{X}^{\tilde{m}} &= i\Gamma_{\tilde{m}}^{ab} \epsilon_a \tilde{\lambda}_b, & \delta \tilde{X}^m &= i\Gamma_m^{ap} \epsilon_a \tilde{\xi}_p, \\
\delta \tilde{\lambda}_a &= i\tilde{F}_{\mu\nu} \sigma^{\mu\nu} \epsilon_a + \Gamma_{\tilde{m}}^{ab} \gamma^\mu \epsilon_b D_\mu \tilde{X}^{\tilde{m}} - \Gamma_{ij}^{ab} \epsilon_b [\tilde{X}^i, \tilde{X}^j], \\
\delta \tilde{\xi}_p &= \Gamma_m^{pa} \gamma^\mu \epsilon_a D_\mu \tilde{X}^m - \Gamma_{ij}^{pa} \epsilon_a [\tilde{X}^i, \tilde{X}^j].
\end{aligned} \tag{4.66}$$

The additional supersymmetry transformation and the fermionic mass term are given by (4.56)–(4.57) with the indices adjusted to the notation of this subsection. From the invariance of the mass-deformed theory we have the mass matrix M_{mn} of four massive bosonic fields,

$$M_{mn} = \mu^2 \text{diag}(1, 1, 1, 1), \quad \text{with } \mu^2 = \mu_5^2 = \mu_6^2 = \mu_7^2 = \mu_8^2. \tag{4.67}$$

As usual, supersymmetry invariance determine the following relations for \tilde{T}_{ijk}

$$\begin{aligned}
\tilde{T}_{ijm} \Gamma_m^{ap} &= \frac{1}{3} \mu_{qq'} (\Gamma_i^{aq} \Gamma_j^{q'p} - \Gamma_j^{aq} \Gamma_i^{q'p}) - \frac{2i}{3} \mu_{pq} \Gamma_{ij}^{aq}, \\
\tilde{T}_{ij\tilde{m}} \Gamma_{\tilde{m}}^{ab} &= \frac{1}{3} \mu_{pq} (\Gamma_i^{ap} \Gamma_j^{qb} - \Gamma_j^{ap} \Gamma_i^{qb}).
\end{aligned} \tag{4.68}$$

These equations leave the nonvanishing components

$$\begin{aligned}
\tilde{T}_{145} &= \frac{1}{3}(\mu_6 + \mu_7), & \tilde{T}_{246} &= \frac{1}{3}(\mu_5 + \mu_7), & \tilde{T}_{347} &= \frac{1}{3}(\mu_5 + \mu_6), \\
\tilde{T}_{127} &= -\frac{1}{3}(\mu_7 + \mu_8), & \tilde{T}_{136} &= -\frac{1}{3}(\mu_5 + \mu_7), & \tilde{T}_{235} &= -\frac{1}{3}(\mu_5 + \mu_8),
\end{aligned} \tag{4.69}$$

where the μ_p 's should satisfy

$$\mu_5 + \mu_6 + \mu_7 + \mu_8 = 0. \tag{4.70}$$

Now we have one massless vector multiplet $(A_\mu^+, \tilde{X}^m, \tilde{\lambda}_a)$ and one massive chiral multiplet $(\tilde{X}^{\tilde{m}}, \tilde{\xi}_p)$. It is also important to note that the relations (4.67) and (4.70) for the mass matrix leads to unique choice for the fermionic mass matrix $\mu_{pq} = \mu \text{diag}(1, 1, -1, -1)$ since the other choices are equivalent to this one up to field redefinitions.

Nonexistence of fuzzy two ellipsoid solution: Substituting the ansatz (4.49) into the vacuum equation (4.63), we rewrite the D - and F -term vacuum equations for $\mathcal{N} = 4$ SYM theory as

$$\begin{aligned}
\alpha_1 \alpha_4 &= 0, & \alpha_1 \alpha_5 - \alpha_4 \mu_5 &= 0, & \alpha_1 \alpha_5 + \alpha_4 \mu_6 &= 0, & \alpha_1 \alpha_5 + \alpha_4 \mu_7 &= 0, & \alpha_1 \alpha_5 - \alpha_4 \mu_8 &= 0, \\
\alpha_4 \alpha_5 - \alpha_1 \mu_5 &= 0, & \alpha_4 \alpha_5 + \alpha_1 \mu_6 &= 0, & \alpha_4 \alpha_5 + \alpha_1 \mu_7 &= 0, & \alpha_4 \alpha_5 - \alpha_8 \mu_8 &= 0.
\end{aligned} \tag{4.71}$$

⁴The index notation for this subsection is: the massive bosonic field indices m, n, \dots run over the range $1, \dots, 4$, while the massless bosonic field indices $\tilde{m}, \tilde{n}, \dots$ are $5, 6, 7$; the massless fermionic field indices a, b, \dots run over the range $1, \dots, 4$; the massive fermionic field indices p, q, \dots run over the range $5, \dots, 8$.

In this case, one easily notices that there is only a trivial solution for which $\alpha_1 = \alpha_4 = 0$ and α_5 is any real number. Thus the nontrivial fuzzy two ellipsoid configuration can not be a classical supersymmetric vacuum solution in $\mathcal{N} = 4$ theory as expected.

5 M-theory Origin of Mass-deformed SYM

In this section we identify the M-theory origin of the flux terms for each of the mass-deformed SYM theories discussed in the previous section. This can be achieved by comparing the supersymmetry preserving flux backgrounds of the previous section to the results of the Higgsing procedure of section 3. We invert the relations in (3.35) and find the constant flux in M-theory in terms of the antisymmetric parameters \tilde{T}_{ijk} (3.33) in type IIA string theory. To be specific this results in the following relations for $T_{AB\bar{C}\bar{D}}$ of (3.30),

$$\begin{aligned} T_{a4\bar{b}\bar{4}} &= i\tilde{T}_{ab4} + \tilde{T}_{a4b+4}, \quad \text{with } \tilde{T}_{a4b+4} = \tilde{T}_{b4a+4}, \\ T_{a4\bar{b}\bar{c}} &= \tilde{T}_{bca+4} - \tilde{T}_{a+4b+4c+4} + i(\tilde{T}_{abc} - \tilde{T}_{ab+4c+4}), \end{aligned} \quad (5.72)$$

where all the other components vanish.

The nonvanishing components of \tilde{T}_{ijk} take different values depending on the number of supersymmetries. Here we list the nonvanishing components of $T_{AB\bar{C}\bar{D}}$ for the three cases separately. For the case of $\mathcal{N} = 1$ supersymmetry, they are

$$\begin{aligned} T_{14\bar{1}\bar{4}} &= \tilde{T}_{145} = -\frac{1}{3}(\mu_4 + \mu_5 + \mu_8), \quad T_{24\bar{2}\bar{4}} = T_{246} = \frac{1}{3}(\mu_3 + \mu_5 + \mu_7), \\ T_{34\bar{3}\bar{4}} &= \tilde{T}_{347} = \frac{1}{3}(\mu_2 + \mu_5 + \mu_6), \quad T_{14\bar{2}\bar{3}} = \tilde{T}_{235} - \tilde{T}_{567} = \frac{1}{3}(\mu_2 + \mu_3 + 2\mu_4 + \mu_6 + \mu_7), \\ T_{24\bar{1}\bar{3}} &= \tilde{T}_{136} + \tilde{T}_{567} = \frac{1}{3}(-\mu_2 - \mu_4 + \mu_6 + \mu_8), \\ T_{34\bar{1}\bar{2}} &= \tilde{T}_{127} - \tilde{T}_{567} = \frac{1}{3}(\mu_3 + \mu_4 - \mu_7 - \mu_8). \end{aligned} \quad (5.73)$$

For the case of $\mathcal{N} = 2$ supersymmetry, they are

$$\begin{aligned} T_{14\bar{1}\bar{4}} &= \tilde{T}_{145} = -\frac{1}{3}(\mu_4 + \mu_5 + \mu_8), \quad T_{24\bar{2}\bar{4}} = T_{246} = \frac{1}{3}(\mu_3 + \mu_5 + \mu_7), \\ T_{34\bar{3}\bar{4}} &= \tilde{T}_{347} = \frac{1}{3}(\mu_5 + \mu_6), \quad T_{14\bar{2}\bar{3}} = \tilde{T}_{235} - \tilde{T}_{567} = \frac{1}{3}(\mu_4 - \mu_5 - \mu_8), \\ T_{24\bar{1}\bar{3}} &= \tilde{T}_{136} + \tilde{T}_{567} = \frac{1}{3}(-\mu_4 + \mu_6 + \mu_8), \\ T_{34\bar{1}\bar{2}} &= \tilde{T}_{127} - \tilde{T}_{567} = \frac{1}{3}(\mu_3 + \mu_4 - \mu_7 - \mu_8), \end{aligned} \quad (5.74)$$

For the case of $\mathcal{N} = 1$ supersymmetry, they are

$$\begin{aligned} T_{14\bar{1}4} = \tilde{T}_{145} &= -\frac{1}{3}(\mu_5 + \mu_8), & T_{24\bar{2}4} = T_{246} &= \frac{1}{3}(\mu_5 + \mu_7), \\ T_{34\bar{3}4} = \tilde{T}_{347} &= \frac{1}{3}(\mu_5 + \mu_6), & T_{14\bar{2}3} = \tilde{T}_{235} &= -\frac{1}{3}(\mu_5 + \mu_8), \\ T_{24\bar{1}3} = \tilde{T}_{136} &= \frac{1}{3}(\mu_6 + \mu_8), & T_{34\bar{1}2} = \tilde{T}_{127} &= -\frac{1}{3}(\mu_7 + \mu_8). \end{aligned} \quad (5.75)$$

The constant flux (5.72) is determined in M-theory and is different from the maximal supersymmetry preserving flux in ABJM theory [6, 7]. Therefore, with this constant flux the maximal $\mathcal{N} = 6$ supersymmetry is not preserved in the ABJM theory. When such flux is turned on in M-theory and the masses of the fermionic and bosonic fields are appropriately chosen, the supersymmetry is partially preserved. In these cases some of the scalar fields remain massless and the corresponding transverse directions may become flat. Unlike the ABJM theory with maximal supersymmetry preserving mass-deformation, the MP Higgsing procedure can be applied to these theories preserving partial supersymmetries. We naturally expect the end result of such procedure will be the mass deformed SYM theories discussed in section 4.

6 Conclusion

In this paper we found supersymmetry preserving mass-deformations of the SYM theory in (2+1)-dimensions and identified their M-theory origin. In achieving this goal, we followed the current trend of deriving supersymmetric Yang-Mills matter theories from the Chern-Simons matter theories via the MP Higgsing procedure. In particular, we considered the ABJM theory in the background of arbitrary constant four-form field strength in the directions transverse to the M2-branes. Actually, for our purpose of generating supersymmetry preserving mass deformations of SYM theories, the WZ-type coupling to the dual seven-form strength is important, while the Higgsing of the corresponding coupling to the constant transverse four-form strength gives either a constant term or higher order α' corrections.

It has already been confirmed that a particular constant transverse four-field strength and the dual-seven form field strength led to the maximal supersymmetry preserving mass-deformation in the context of the ABJM theory. In this paper we show that the MP Higgsing of the original ABJM theory without mass-deformation reduces to the (2+1)-dimensional $\mathcal{N} = 8$ SYM theory without any extra decoupled sector in contrast to the claim of an earlier work on the same subject [21]. The result suggests that the corresponding reduction of the maximal supersymmetric mass-deformed ABJM theory might give some known mass deformed SYM theory. This naive expectation did not work because in this case the scalar potential does not contain flat direction and then one could not

take the limit of large vacuum expectation value in pursuing the Higgsing procedure. We instead began with the ABJM theory deformed by a WZ-type coupling to a constant seven-form field strength which is dual to an arbitrary constant four-form field strength in the transverse direction. This deformation breaks supersymmetry but leaves us with some flat directions. Application of the MP Higgsing procedure to the ABJM theory deformed by this WZ-type coupling led to Yang-Mills matter theories including a Myers coupling to a five-form gauge field with constant six-form field strength. By different choices of the nonvanishing components of the five-form gauge field and appropriate identification of the corresponding masses for the fermionic and bosonic fields, we obtained SYM theories with $\mathcal{N} = 1$, $\mathcal{N} = 2$, and $\mathcal{N} = 4$ supersymmetries. We solved the vacuum equations for each of these theories and found the fuzzy two ellipsoid solutions in the first two cases while in the third case the equations support only the trivial solution. The obtained fuzzy two ellipsoid solutions confirm that the D2-branes system polarizes into a D4-branes system with the extra dimensions warping the two ellipsoid when we turn on the mass terms for the matter fields [23].

Finally, we used the values of the nonvanishing components of the R-R five-form and determined the corresponding six-form gauge field in M-theory. This may identify a possible M-theory origin of the supersymmetry preserving mass-deformations of the SYM theories. It is interesting to employ the supersymmetry completion to find the appropriate quadratic mass-deformations in the ABJM theory and figure out which of the supersymmetries of the theory are preserved despite of such deformation. One can then apply the MP Higgsing procedure to the reduced supersymmetric theories and expect to reproduce the mass-deformed SYM theories we obtained in this paper. These points and the issue concerning the dual gravity [26, 27, 28] of the reduced supersymmetric theories will be reported in a separate work [29].

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